

Całki I

Zad.1. Obliczyć całki.

$$(1) \int x^2(1-x)dx$$

$$(2) \int x^{-2}dx$$

$$(3) \int \frac{dx}{10x}$$

$$(4) \int \frac{dx}{3x^4}$$

$$(5) \int (\frac{3}{x^2} + \frac{4}{x^3})dx$$

$$(6) \int \frac{(3-x)^2}{x^3}dx$$

$$(7) \int \frac{1-x^2}{x^4}dx$$

$$(8) \int \frac{x^4-4}{x^2+2}dx$$

$$(9) \int x\sqrt{x}dx$$

$$(10) \int x\sqrt[4]{x^3}dx$$

$$(11) \int \sqrt{x}(\frac{1}{x} + x)dx$$

$$(12) \int \sqrt{x}\sqrt{x}dx$$

$$(13) \int \frac{1}{\sqrt[3]{x^2}}dx$$

$$(14) \int \frac{x^2-x-2}{\sqrt[3]{x^2}}dx$$

$$(15) \int \frac{(x+\sqrt{x})(\sqrt{x}+\sqrt[4]{x})(\sqrt{x}-\sqrt[4]{x})}{x}dx$$

$$(16) \int \frac{\sqrt{x+2}\sqrt{x+1}}{x}dx$$

$$(17) \int \frac{\sqrt[3]{x}-1}{\sqrt[6]{x}-1}dx$$

$$(18) \int (4^x+2^{-x})dx$$

$$(19) \int 5^xe^xdx$$

$$(20) \int 3^x \cdot 2^{-2x}dx$$

$$(21) \int \frac{27^{5x} \cdot 8^{3x}}{2^{11x} \cdot 9^{7x}}dx$$

Zad.2. Obliczyć całki.

$$\begin{array}{lll}
(1) \int x \cos x dx & (6) \int \ln(3x) dx & (11) \int x^3 e^x dx \\
(2) \int x^2 \sin x dx & (7) \int \ln^2 x dx & (12) \int x^2 e^{-x} dx \\
(3) \int x \ln x dx & (8) \int x^6 \ln x dx & (13) \int x^2 \cdot 2^x dx \\
(4) \int x^3 \ln(2x) dx & (9) \int 16(x \ln x)^3 dx & \\
(5) \int \ln x dx & (10) \int x e^x dx & (14) \int x \operatorname{arctg} x dx
\end{array}$$

Zad.3. Obliczyć całki.

$$\begin{array}{ll}
(1) \int \left(\frac{1}{5}x - 3\right)^{14} dx & (12) \int \frac{\ln x + 1}{\ln x \cdot x} dx \\
(2) \int \frac{1}{(2x - 1)^5} dx & (13) \int \frac{\ln x + 2}{(1 + \ln x)x} dx \\
(3) \int \sqrt{\frac{1}{3}x - 1} dx & (14) \int \frac{4x}{2x^2 + 3} dx \\
(4) \int \frac{a}{bx + c} dx^1 & (15) \int \frac{x^3}{x^4 + 2} dx \\
(5) \int \sqrt[3]{\left(\frac{1}{5}x - 4\right)^2} dx & (16) \int \frac{6x^2}{\sqrt{x^3 + 1}} dx \\
(6) \int \frac{1}{\sqrt{1 - 4x}} dx & (17) \int (2x - 1)\sqrt{x^2 - x} dx \\
(7) \int \frac{2}{\sqrt[4]{2x + 1}} dx & (18) \int \frac{dx}{\sqrt{x} + 1} \\
(8) \int 2x^3 e^{x^4} dx & (19) \int \tan x dx \\
(9) \int 2 \cos x \cdot e^{\sin x} dx & (20) \int \cot \frac{x}{2} dx \\
(10) \int 2 \sin x (\cos x - 1) dx & (21) \int \frac{3e^x}{e^x + 2} dx \\
(11) \int \frac{dx}{x \ln x} & (22) \int \frac{x}{4 - x^2} dx
\end{array}$$

¹Przy założeniu, że $a, b \neq 0$.

$$(23) \int x^2 \sqrt[3]{x^3 + 1} dx$$

$$(24) \int \sin^2 x \cos x dx$$

$$(25) \int 4 \sin x \cos^7 x dx$$

$$(26) \int x \sin(x^2) dx$$

$$(27) \int \frac{\operatorname{arctg}^2 x - \operatorname{arctg} x}{x^2 + 1} dx$$

$$(28) \int \frac{\cos(\ln x)}{x} dx$$

$$(29) \int \frac{dx}{x^2 + 9}$$

$$(30) \int \frac{1}{1 - \sin x} dx$$

$$(31) \int \frac{1}{\cos x - 1} dx$$

$$(32) \int \frac{\cos^2 \sqrt[3]{x} \sin \sqrt[3]{x}}{\sqrt[3]{x^2}} dx$$

Zad.4. Obliczyć.

$$(1) \int 2x \ln(x^2 + 2) dx$$

$$(2) \int e^{\sqrt{x}} dx$$

$$(3) \int \frac{1}{\sqrt[3]{x}} \sin \sqrt[3]{x} dx$$

$$(4) \int \operatorname{arctg} x dx$$

$$(5) \int \frac{\operatorname{arctg}(\ln x)}{x} dx$$

$$(6) \int \frac{\operatorname{arctg}^2 x - 2}{(\operatorname{arctg} x + 3)(x^2 + 1)} dx$$

$$(7) \int (4x^7 + 4x^3) e^{x^4 + 1} dx$$

$$(8) \int \frac{dx}{\sin^4 x \cos^2 x}$$

$$(9) \int \frac{dx}{\cos x}$$

$$(10) \int \frac{dx}{\sqrt{x} \cdot \sin(\sqrt{x} + 1)}$$

$$(11) \int \sin x \cdot \ln(\cos^2 x + 1) dx$$

$$(12) \int \frac{x - 2\sqrt{x} + 1}{x + \sqrt{x}} dx$$

$$(13) \int \frac{1}{e^x + e^{-x}} dx$$

$$(14) \int \frac{e^{2x} + 2}{e^x + 1} dx$$

$$(15) \int e^x \cos x dx$$

Zad.5. Obliczyć całki z funkcji wymiernych.

$$(1) \int \frac{x + 2}{x - 3} dx$$

$$(3) \int \frac{x^2 dx}{x - 1}$$

$$(5) \int \frac{2x^3 - 3x^2 + 4}{x - 3} dx$$

$$(2) \int \frac{2x + 3}{2x + 5} dx$$

$$(4) \int \frac{x^3 + x + 1}{x - 1} dx$$

$$(6) \int \frac{2x^3 + 3x^2 - 2x + 1}{2x + 3} dx$$

$$\begin{array}{lll}
(7) \int \frac{x-1}{(x+2)(x+4)} dx & (21) \int \frac{2x dx}{(x+4)^2} & (34) \int \frac{x^2-11x+6}{x^3+2x^2-4x-8} dx \\
(8) \int \frac{10}{(x+3)(x-7)} dx & (22) \int \frac{4x dx}{4x^2-12x+9} & (35) \int \frac{x^2+6x-5}{x^3+x^2+x+1} dx \\
(9) \int \frac{22-x}{(x-1)(x-8)} dx & (23) \int \frac{2x^3-2}{x^2+2x+1} dx & (36) \int \frac{2}{x^3+4x^2+4x} dx \\
(10) \int \frac{x}{x^2-7x+6} dx & (24) \int \frac{dx}{x^2-x+1} & (37) \int \frac{2(x^2+1)}{x^3-4x} dx \\
(11) \int \frac{2x+1}{x^2+6x+8} dx & (25) \int \frac{x dx}{x^2-2x+2} & (38) \int \frac{x^4-3x^2-2x-8}{x^3-4x} dx \\
(12) \int \frac{1}{x^2-3x} dx & (26) \int \frac{2x+3}{x^2+4x+5} dx & (39) \int \frac{17x^2-10x+1}{6x^3-11x^2+6x-1} dx \\
(13) \int \frac{x^2-2x+2}{x^2-3x+2} dx & (27) \int \frac{x^2-3}{x^2-6x+13} dx & (40) \int \frac{x-1}{x^4-1} dx \\
(14) \int \frac{x^3+8}{x^2-16} dx & (28) \int \frac{x^4+1}{x^2-2x+2} dx & (41) \int \frac{2x^3+3x^2+13x+4}{x^4+4x^3} dx \\
(15) \int \frac{x dx}{x^2-11x+30} & (29) \int \frac{dx}{(x-1)(x-2)(x-3)} & (42) \int \frac{3x^2-8x+3}{x^4+3x^3} dx \\
(16) \int \frac{dx}{(x+3)^2} & (30) \int \frac{2x^2-5x-6}{(x^2+x-2)(x-2)} dx & (43) \int \frac{x^2-5x+5}{x^4-x^3} dx \\
(17) \int \frac{dx}{x^2-6x+9} & (31) \int \frac{5x^2-9}{2x^3+3x^2} dx & (44) \int \frac{x^3-2x^2+x-2}{x^4-x^2} dx \\
(18) \int \frac{x+2}{(x+1)^2} dx & (32) \int \frac{2x-10}{x^3-6x^2+10x} dx & (45) \int \frac{3x^3-6x^2-20x-24}{x^4-16} dx \\
(19) \int \frac{x^2+1}{(x+2)^2} dx & (33) \int \frac{3x^2-8x+4}{x^3-3x^2+3x-1} dx & (46) \int \frac{-2x^2+6x+1}{x^4+5x^2+4} dx \\
(20) \int \frac{dx}{(3x+5)^2} & &
\end{array}$$