Liczby zespolone

Zad.1. Wykonać działania. Dla wyznaczonych liczb zespolonych z wyznaczyć $\Re(z)$, $\Im(z)$, $\bar{z}$ oraz $|z|$.

(1) $(2+5i)(3+i)$

(2) $(5+3i)(2i-1)$

(3) $(1-i)^6$

(4) $\frac{2}{i-1}$

(5) $\frac{2-i}{3+4i}$

(6) $\frac{4+2i}{3-5i}$

(7) $\frac{(2+3i)(1+2i)}{3-i}$

(8) $\frac{(1-i)(2+3i)}{-3+i}$

(9) $\frac{20-5i}{(1-2i)(i+3)}$

(10) $\frac{(1+4i)(3-i)}{(1-3i)(2+i)}$

(11) $\frac{(6-i)(2-3i)}{(3+2i)(2+3i)}$

(12) $\frac{2i(i+3)+(i-1)^2}{2(2-i)^2}$

(13) $\frac{2-i}{1-i}+\frac{4+i}{1+i}$

(14) $\frac{10-5i}{2+i}+\frac{17i-7}{5-i}$

(15) $\frac{2i^9-(i+5)(5i-1)}{(1+i)^2}$

(16) $\frac{4i(2i-1)-(1+3i)^2}{(i-1)^4}$

(17) $\frac{1}{2i-3}-\frac{i}{3i-2}$

(18) $\frac{(i+1)^4}{2-i}$

(19) $\frac{25}{4-3i}+i(i-5)(1-i)$

(20) $(3-2i)^2+(1+2i)^3$

(21) $\left(\frac{2-i}{1+i}\right)^2+5\left(\frac{1-i}{2+i}\right)^2$

(22) $\frac{4+2i}{1+2i-\frac{1+3i}{3-i}}+\frac{1-2i}{2+i}$

Zad.2. Wykonać obliczenia.

(1) $\mathcal{R}e(z\cdot i)+\mathcal{I}m(\frac{z}{i-1})$ dla $z=4+3i$,

(2) $\mathcal{R}e(\frac{(i+1)z}{i-2})+\mathcal{I}m(\frac{z^2}{1-i})$ dla $z=-1-2i$,

(3) $\mathcal{R}e(\frac{i}{z})-\mathcal{I}m(\frac{z-2}{i(z-1)})$ dla $z=5i+1$,

(4) $\frac{|z-1|^2+\mathcal{R}e(\frac{z(i+1)}{i})}{|z+1|^2+\mathcal{I}m(\frac{i\cdot z}{i+1})}$ dla $z=-2+4i$,

(5) $\frac{\mathcal{I}m(\frac{i}{z})+|\frac{z+1}{z-1}|^2+\mathcal{R}e(\frac{z}{i})}{\mathcal{R}e(\frac{i}{z})+|\frac{z-i}{z+i}|^2+\mathcal{I}m(\frac{z}{i})}$ dla $z=2-i$.

Zad.3. Wyznaczyć $x,y\in\mathbb{R}$ tak, aby spełnione były poniższe równania.

(1) $(1+i)(x-y)+(1-i)(x+y)=2+4i$

(2) $(2-i)(x+2y)-(2i-1)(y-ix)=-5$

(3) $(3-i)(x+yi)-(1+2i)(y-xi)=3-16i$

(4) $(5-i)(x+2iy)+(2i+1)(y-5ix)=3+78i$

Zad.4. Rozwiązać równania.

(1) $z^2=2\bar{z}$

(2) $(z+i)(\bar{z}-1)=z-2i\cdot\bar{z}$

(3) $\bar{z}+\frac{2}{z}=3$

(4) $(z-1)^2+(\bar{z}-2i)^2=0$

(5) $\Re(z)(2z-i)=2\bar{z}+4$

(6) $z^2+2|z|^2+(1-i)z-2(1+i)\bar{z}=2i-2$

Zad.5. Przedstawić podane liczby w postaci trygonometrycznej.

(1) $\frac{2i+1}{2-i}$

(2) $\frac{4-10i}{5+2i}$

(3) $\frac{15-5i}{4-3i}-2$

(4) $\frac{-5-4i}{-4+5i}-1$

(5) $\frac{16-3i}{7+2i}-3$

(6) $\frac{\sqrt{3}-2+(1+2\sqrt{3})i}{1+2i}$

(7) $\frac{1-3\sqrt{3}i}{2i-\sqrt{3}}$

(8) $\frac{-\sqrt{3}-5i}{\sqrt{3}-2i}$

(9) $\frac{11+2\sqrt{3}i}{4-\sqrt{3}i}-3$

(10) $2i-\frac{3\sqrt{3}-i}{\frac{1}{2}-\frac{\sqrt{3}i}{2}}$

Zad.6. Obliczyć.

(1) $\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)^{21}$

(2) $\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)^{34}$

(3) $(i-1)^{12}$

(4) $(i+1)^{-4}$

(5) $(\sqrt{3}+i)^6$

(6) $\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^8$

(7) $(3\sqrt{3}-3i)^4$

(8) $(1-\sqrt{3}i)^{10}$

(9) $(-1+\sqrt{3}i)^7$

(10) $\frac{(i+1)^{10}}{(2i)^6}$

(11) $\frac{(\sqrt{3}-i)^8}{(1+i)^5}$

(12) $\left(\frac{1-i}{\sqrt{3}-i}\right)^5$

(13) $\frac{(i-1)^{27}(\sqrt{3}-i)^5}{(i+1)^{28}(\sqrt{3}+i)^4}$

(14) $\left(\frac{5-i}{3+2i}\right)^9$

Zad.7. * Wyznaczyć moduł i argument podanych liczb zespolonych.

(1) $\cos\phi+i\sin\phi$

(2) $-\cos\phi+i\sin\phi$

(3) $-\cos\phi-i\sin\phi$

(4) $\sin\phi-i\cos\phi$

(5) $\frac{1+i\operatorname{tg}\phi}{1-i\operatorname{tg}\phi}$

(6) $\left(\frac{1-i\operatorname{tg}\phi}{1+i\operatorname{tg}\phi}\right)^n$

Zad.8. * Korzystając ze wzoru de Moivre'a wyrazić podane funkcje za pomocą $\sin\alpha$ i $\cos\alpha$.

(1) $\sin 3\alpha$

(2) $\cos 4\alpha$

Zad.9. Obliczyć pierwiastki zespolone.

(1) $\sqrt[3]{-27}$
(2) $\sqrt{8}$
(3) $\sqrt[4]{-1}$
(4) $\sqrt[6]{1}$
(5) $\sqrt[3]{1+i}$
(6) $\sqrt[3]{8i}$
(7) $\sqrt{-2i}$
(8) $\sqrt[4]{-4i}$
(9) $\sqrt[3]{\sqrt{3}-i}$
(10) $\sqrt{\frac{2}{\sqrt{3}i+1}}$
(11) $\sqrt{\frac{1-\sqrt{3}i}{1-i}}$
(12) $\sqrt[3]{\frac{1+\sqrt{3}i}{1+i}}$
(13) $\sqrt[3]{\frac{1+\sqrt{3}+(1-\sqrt{3})i}{2(1-i)}}$
(14) $\sqrt[8]{1}$
(15) $\sqrt[6]{-1}$

Zad.10. Rozwiązać równania.

(1) $z^2-4z+8=0$
(2) $z^2+6z+11=0$
(3) $9z^2-6z+4=0$
(4) $\frac{z^2-1}{2z+1}=z$
(5) $\frac{3z-1}{z-2}=\frac{2z+3}{z+1}$
(6) $iz^2+(i-1)z-3=0$
(7) $iz^2+4z-2iz-2-3i=0$
(8) $(z-1-i)^2=2i$
(9) $\left(\frac{z}{1-i}\right)^2=2-2\sqrt{3}i$
(10) $\left(\frac{iz}{2+i}\right)^3=-8$
(11) $z^5-iz^2=0$
(12) $z^7+iz^4=0$
(13) $(z^4-1)(z^3+i)=0$
(14) $(z-1)^2=\left(\frac{1-i}{1+i}\right)^3$
(15) $z^6+2z^3+2=0$
(16) $\left(\frac{z-1}{i}\right)^3=\left(\frac{1-\sqrt{3}i}{2i}\right)^4$
(17) $z^3=(2+i)^3$
(18) $z^4=(3-i)^4$

Zad.11. Zaznaczyć na płaszczyźnie Gaussa podane zbiory.

(1) $\left\{z\in\mathbb{C}:\ \arg(z)=\frac{5\pi}{4},\ \Re(z)\geq 1\right\}$
(2) $\{z\in\mathbb{C}:\ |z|=3,\ \Im(z)<-1\}$
(3) $\{z\in\mathbb{C}:\ 1\leq |z|\}$
(4) $\left\{z\in\mathbb{C}:\ 2\leq |z|\leq 3, \frac{\pi}{4}\leq \arg(z)\leq\frac{3\pi}{4}\right\}$
(5) $\{z\in\mathbb{C}:\ |z-2|>2, \Re(z)>2\}$
(6) $\{z\in\mathbb{C}:\ |z-5i|>5, \Re(z)<3\}\cup\{z\in\mathbb{C}:\ |z-5i|<5, \Re(z)\geq 3\}$
(7) $\left\{z\in\mathbb{C}:\ \Re(z(2+i))=\Im(\frac{5z}{1-2i})\right\}$
(8) $\{z\in\mathbb{C}:\ \mathcal{R}e[(1+i)\mathcal{I}m(z(2i+3)-3)+1]=\Im[(2-i)\Re(z(3+i)+1)-2]\}$
(9) $\left\{z\in\mathbb{C}:\ \Im\left(\frac{1}{z}\right)\geq\mathcal{R}e\left(\frac{1}{\bar{z}}\right)\right\}$
(10) $\left\{z\in\mathbb{C}:\ \frac{\pi}{2}<\arg(2z)<\frac{3\pi}{4}\right\}$
(11) $\left\{z\in\mathbb{C}:\ \frac{\pi}{2}<\arg(iz)<\frac{7\pi}{4}\right\}$

(12) $\{z \in \mathbb{C}:\ 0 < \arg(z^3) < \pi,\ 1 < |z| < 3\}$

(13) $\{z \in \mathbb{C}:\ \Re(z) + |z| < \Im(z)\}$