Pochodna

Zad.1. Na podstawie definicji obliczyć pochodną funkcji f w podanym punkcie x_0.

(1) $f(x) = 2x - x^2,\ x_0 = 1$
(2) $f(x) = x^2 - 7x,\ x_0 = 0$
(3) $f(x) = x^3,\ x_0 = 1$
(4) $f(x) = -2x^3 + x,\ x_0 = -1$
(5) $f(x) = -\sqrt{x+2},\ x_0 = 2$
(6) $f(x) = \sqrt{1+2x},\ x_0 = 4$
(7) $f(x) = 3 + 2\sqrt{4x-1},\ x_0 = 2\frac{1}{2}$
(8) $f(x) = \frac{1}{x^2},\ x_0 = 1$
(9) $f(x) = \frac{2}{x^3},\ x_0 = -1$
(10) $f(x) = \sqrt{x^2-1},\ x_0 = -3$
(11) $f(x) = \sqrt{2x^2+1},\ x_0 = 2$
(12) $f(x) = \sin x,\ x_0 = \frac{\pi}{2}$
(13) $f(x) = \cos x,\ x_0 = \frac{\pi}{2}$
(14) $f(x) = \sin x^2,\ x_0 = 0$
(15) $f(x) = \sin\sqrt{x},\ x_0 = 0$

Zad.2. Wyznaczyć pochodne podanych funkcji.

(1) $y = (3 - x^4)^2$
(2) $y = 1 - \frac{4}{x^6} + \frac{3}{x^7}$
(3) $y = \frac{x^2+4}{x}$
(4) $y = \frac{x-3}{x^2}$
(5) $y = (\sqrt[4]{x} - \frac{1}{2})^2$
(6) $y = \frac{2}{\sqrt[6]{x^5}} - \frac{x}{3} + \frac{\sqrt[4]{x^3}}{2}$
(7) $y = \frac{x^5+x^3+x}{\sqrt{x}}$
(8) $y = \sqrt[3]{\sqrt{x}x}$
(9) $y = \sqrt[5]{\sqrt[4]{x}x}$
(10) $y = x\cos x$
(11) $y = x^2\sin x + \operatorname{tg} x$
(12) $y = \frac{\sin x}{x^3}$
(13) $y = \sqrt{x}\ln x$
(14) $y = \frac{1-\sin x}{1+\cos x}$
(15) $y = \frac{1}{\sqrt{x}+1}$
(16) $y = \frac{\sqrt[3]{x}}{x-1}$
(17) $y = \sqrt{\sin x + 2}$
(18) $y = \sqrt[3]{2x-1}$
(19) $y = (\frac{1}{100}x^3 - 5)^{100}$
(20) $y = x(x^5 + 2x - 1)^3$
(21) $y = (x-1)\sqrt{3x+1}$
(22) $y = x^2\sqrt[3]{4x+1}$
(23) $y = \operatorname{ctg}\frac{x^5}{5}$
(24) $y = 2\operatorname{arctg}\frac{x}{4} - x^{17}$
(25) $y = \operatorname{tg}\frac{1}{x}$
(26) $y = \frac{1}{\cos^3 x + 1}$
(27) $y = \cos(1 + 2x^2)$
(28) $y = \sin(x + \cos x)$
(29) $y = x^2\operatorname{arctg} x - \operatorname{tg} 5x$
(30) $y = x\ln x - \cos(x^6)$
(31) $y = \sqrt{x}\ln(x+1) - \frac{\sqrt{x}}{x+1}$
(32) $y = \ln(x^2+1)$
(33) $y = 2\ln(x^7 + x - 1)$

(34) $y = 2x^3 \ln(x^2 + 2x + 1)$

(35) $y = \ln(\frac{x-1}{x^2})$

(36) $y = \ln\sqrt{x+3}$

(37) $y = \sqrt{\ln(x-6)}$

(38) $y = \ln(\sqrt{x^2+8} - x)$

(39) $y = x \arccos\frac{x}{2}$

(40) $y = x^5 \ln^5 x$

(41) $y = (\frac{1}{2})^{\sin x}$

(42) $y = \sqrt{2x - \sqrt{x}}$

(43) $y = \ln\sin x$

(44) $y = \sin\ln x$

(45) $y = 4^x \cdot 5^{x^2}$

(46) $y = e^{\sin^2 x}$

(47) $y = \operatorname{arctg}\frac{x+1}{x-1}$

(48) $y = \operatorname{arctg}\frac{2x+3}{3x-2}$

(49) $y = \operatorname{arctg}\frac{2\ln x - 1}{\ln x + 1}$

(50) $y = \operatorname{arctg}\frac{\ln x - x}{x + 4\ln x}$

(51) $y = \ln\frac{\operatorname{arctg} x + 3}{\operatorname{arctg} x - 1}$

(52) $y = x^x$

(53) $y = (2x)^{2x}$

(54) $y = (x+1)^{x+2}$

(55) $y = (x + \ln x)^{x - \ln x}$

(56) $y = (\sin x)^{4\cos x}$

(57) $y = \log_2\left(\operatorname{tg}\frac{5^x}{x}\right)$

(58) $y = \frac{\arcsin(1-x)\cdot\ln(x^3)}{\cos x}$

(59) $y = \frac{e^{x-2^x}\cdot\cos\frac{1}{x} - \ln x^{-4}}{\arccos\sqrt{x}}$

(60) $y = \frac{\ln(\cos^4 x + \sin^4 x)}{\cos^2(\ln x) + \sin(\ln^2 x)}$

(61) $y = \frac{4^{x^2-x}\cdot\operatorname{tg}^2(e^3 - \ln x)}{x\arcsin\frac{1}{x}}$

(62) $y = \frac{\ln(x^6\operatorname{arctg}^2 x)\cdot\operatorname{arctg}(x^2\ln^6 x)}{\sqrt{x^2+1}}$

(63) $y = \frac{\sinh 5^x\sqrt{\ln x + 1} - e^7\operatorname{arctg}\frac{1}{x}}{\arcsin(x2^x)}$

Zad.3. Obliczyć pochodne drugiego rzędu podanych funkcji.

(1) $y = \ln(\sin x)$

(2) $y = \sin(\ln x)$

(3) $y = \ln(2\cos x - 1)$

(4) $y = 2x\ln x$

(5) $y = \sqrt{x}\ln x$

(6) $y = 5^x - 5^{-x}$

(7) $y = \operatorname{arctg} 2x$

(8) $y = \ln^3 x$

(9) $y = \ln(x^2+1)$

(10) $y = \ln^2(x^4+1)$

(11) $y = xe^x$

(12) $y = x^5 e^{2x}$

(13) $y = xe^{x^3}$

(14) $y = \sqrt{2x^2 - 1}$

(15) $y = \sqrt[3]{x^3 + 2x^2 + 1}$

(16) $y = \sin(x^2)$

(17) $y = \cos^2 x$

(18) $y = \frac{\ln x}{x^2}$

(19) $y = \frac{x}{\ln x}$

(20) $y = \frac{2x+1}{x-2}$

(21) $y = \frac{x^3+1}{x-1}$

(22) $y = x\arcsin x$

(23) $y = \arccos x^4$

(24) $y = x\operatorname{arctg} x$

(25) $y = x\operatorname{arctg}\sqrt{x}$

Zad.4. Obliczyć pochodne trzeciego rzędu podanych funkcji.

(1) $y = \sin^2 x$

(2) $y = x^3\ln x$

(3) $y = \frac{\ln x}{x^2}$

(4) $y = \ln^2 x$

(5) $y = 2\operatorname{arcctg} x$

(6) $y = x\cos x$

(7) $y = 2x\sin 3x$

(8) $y = xe^{2x}$

(9) $y = x^3 e^x$

Zad.5. Wyznaczyć pochodne podanych funkcji parametrycznych.

(1) $\begin{cases} x(t) = 2t^2 - 3 \\ y(t) = t^3 - t^2 \end{cases}$ (4) $\begin{cases} x(t) = \cos^3 t \\ y(t) = \sin^3 t \end{cases}$ (7) $\begin{cases} x(t) = \ln(\cos t) \\ y(t) = \ln(\sin t) \end{cases}$

(2) $\begin{cases} x(t) = \ln t \\ y(t) = \ln(t^2 + 1) \end{cases}$ (5) $\begin{cases} x(t) = \arcsin t \\ y(t) = \arcsin t^2 \end{cases}$ (8) $\begin{cases} x(t) = e^{\sin^2 t} \\ y(t) = e^{\cos^2 t} \end{cases}$

(3) $\begin{cases} x(t) = \sqrt{1 - t^3} \\ y(t) = \frac{1}{\sqrt{1-t^3}} \end{cases}$ (6) $\begin{cases} x(t) = \frac{t}{t^2+1} \\ y(t) = \frac{1}{t^2+1} \end{cases}$

Zadania dodatkowe

Zad.1. Naszkicować wykres funkcji f. Czy jest to funkcja różniczkowalna?

(1) $f(x) = \begin{cases} x & x < 1 \\ x^3 & x \geq 1, \end{cases}$

(2) $f(x) = \begin{cases} \frac{1}{2}x & x \leq 0 \\ x^2 & x > 0, \end{cases}$

(3) $f(x) = \begin{cases} \frac{1}{2}x^3 & x < 0 \\ x^2 & x \geq 0. \end{cases}$

Zad.2. Wyznaczyć parametry a, b tak, aby funkcja f była różniczkowalna w $\mathbb{R}$:

$$f(x) = \begin{cases} ax + b & \text{dla } x < 1 \\ ax^2 + x + 2b & \text{dla } x \geq 1. \end{cases}$$