Własności funkcji

Zad.1. Wyznaczyć dziedzinę podanych funkcji.

(1) $f(x) = \frac{\sqrt{1-x^2}}{2^x-4}$

(2) $f(x) = \frac{\log x \cdot \sqrt{\operatorname{tg} x}}{x^2+3x+5}$

(3) $f(x) = \frac{\sqrt{\log(2x^2-8)}}{3-3^x}$

(4) $f(x) = \ln\sqrt{1-\sin 2x}$[1]

(5) $f(x) = \ln\sqrt{\frac{1}{2}-\cos 3x}$

(6) $f(x) = \sqrt{\ln\frac{2x}{x-1}}$

(7) $f(x) = \sqrt{\sqrt{2x}-3}$

(8) $f(x) = \sqrt{5-\sqrt{5-x}}$

(9) $f(x) = \arccos(\frac{3x-1}{2})$

(10) $f(x) = \frac{4-x}{\arcsin\frac{x}{4}}$

(11) $f(x) = \frac{\sqrt{2x-1}}{2+\arccos\frac{x+1}{4}}$

(12) $f(x) = 10^{\frac{2}{3\arcsin x^2}}$

(13) $f(x) = \frac{\arcsin(2-x)}{x^2-4x+4}$

(14) $f(x) = \frac{\arcsin\frac{x+1}{13}-\arccos\frac{x-2}{11}}{x^{12}}$

(15) $f(x) = \frac{\arccos(\sqrt{x}-1)}{9x^2-6x+1}$

(16) $f(x) = \sqrt{\arccos(2x+1)}$

(17) $f(x) = \arcsin\sqrt{\frac{x+1}{x}}$

(18) $f(x) = \frac{\arccos\sqrt{\frac{x-2}{x+3}}}{\sqrt{2x+4}}$

(19) $f(x) = \arcsin^3(\ln x + 5)$

(20) $f(x) = \arccos\left(\frac{\ln^3 x-14}{13}\right)$

(21) $f(x) = \log_2(\arcsin\frac{x}{3})$

(22) $f(x) = \frac{1}{\arccos(1-\ln x)}$

(23) $f(x) = \arctan(12 - \frac{2x}{\sqrt{x^2+1}})$

(24) $f(x) = \operatorname{arc\,ctg}(1-\ln^7 x^3)$

(25) $f(x) = \arccos(x^2-4)$

(26) $f(x) = \arcsin(x^2-\frac{1}{2})$

(27) $f(x) = \arcsin\frac{x-3}{x-4}$

(28) $f(x) = \arccos\frac{2-x}{x-5}$

(29) $f(x) = \sqrt{2x+1}\arcsin\frac{x-2}{x}$

(30) $f(x) = \sqrt{\arcsin(x+\frac{1}{3})}$

(31) $f(x) = \frac{\sqrt{x+3}}{\arcsin\frac{x-2}{6}}$

(32) $f(x) = \frac{2}{\sqrt{3x+1}} + \arctan\frac{1}{x}$

(33) $f(x) = \frac{\arcsin\frac{x^3+2}{x^3}}{x^3+7x^2}$

(34) $f(x) = \arccos(\sin^3 x + 1)$

(35) $f(x) = \arccos\frac{x-\sqrt{x}+1}{x+\sqrt{x}+1}$

(36) $f(x) = \arcsin\frac{x^2-4}{x^2-1}$

(37) $f(x) = \frac{\ln(\sqrt{4x+1}-2)}{2\operatorname{tg}^2\left(\frac{\arcsin x+1}{\arcsin x}\right)+5}$

[1] $\ln a = \log_e a$, gdzie $e \approx 2,718$.

Zad.2. Opisać analitycznie i naszkicować w układzie współrzędnych dziedziny podanych funkcji.

(1) $f(x,y) = \frac{1}{\sqrt{2y-3x}}$

(2) $f(x,y) = \frac{\sqrt{y+1}}{\ln(x+2)}$

(3) $f(x,y) = \arcsin\frac{x}{2} + \frac{1}{\sqrt{y^2-1}}$

(4) $f(x,y) = \sqrt{1-x^2-y^2}$

(5) $f(x,y) = e^{\sqrt{x-y}} + \arcsin(y-x^2)$

(6) $f(x,y) = \frac{e^{\sqrt{x}}}{y-2x} \cdot \ln(x^2+y^2-1)$

(7) $f(x,y) = \arccos\frac{2x-3y}{12} + \frac{1}{\sqrt{x-\sqrt{4-y}}}$

(8) $f(x,y) = \arcsin\frac{y}{x^2} + \frac{\ln(\sqrt{x}+y-2)}{\sqrt{x-1}\cdot e^{\sqrt{y+1}}}$

(9) $f(x,y) = (\sqrt{x+y-2} + \sqrt{2y-x-3}) \cdot \ln(x^2+y^2-1)$

(10) $f(x,y) = \frac{\arcsin\frac{x-y}{4}}{\sqrt{2x-y-1}-\sqrt{2y-x-1}}$

(11) $f(x,y) = \frac{\arctan(2^{\sqrt{x^3-8y}}\cdot\ln(1-\sqrt{x-1}))}{\sqrt{64-x^2-y^2}}$

(12) $f(x,y) = \log_{\sqrt{xy}}(\frac{(x-1)(y-1)}{(x+y-1)(x+y+1)})$

(13) $f(x,y) = \sqrt{(\sqrt{x}+\sqrt{y})^2+1} \cdot \ln(\sqrt{4-x^2}+\sqrt{9-y^2})$

(14) $f(x,y) = \arccos^3\frac{1}{\sqrt[3]{x}-1} + \arcsin^7\frac{1}{1-\sqrt[5]{x}} + \ln(4-x^2-y^2)$

(15) $f(x,y) = \arcsin(\ln(y+1)-2x) + \sqrt[4]{x^3-1}\cdot e^{\sqrt{y^5+1}}$

(16) $f(x,y) = \arcsin(\ln(x^3+y+1)) + \ln(x^2+y^2-4)\cdot\frac{\arctan\frac{x+y}{1-x^2}}{\sqrt{16-x^2-y^2}}$

Zad.3. Obliczyć.

(1) $\arccos(-\frac{1}{2}) + \arcsin 0$

(2) $3\arcsin 1 - 2\arccos 0$

(3) $4\arctan 1 - \operatorname{arc\,ctg}(-1)$

(4) $\frac{1}{2}\arccos\frac{\sqrt{3}}{2} + \arctan(-\sqrt{3})$

(5) $\operatorname{arc\,ctg}\sqrt{3} - 3\arcsin\frac{\sqrt{2}}{2}$

(6) $2\arccos(-\frac{1}{2}) - \arctan 1$

(7) $\arctan\frac{\sqrt{3}}{3} - 2\arcsin\frac{1}{2}$

(8) $\left(3\arccos\frac{\sqrt{3}}{2} + \pi\right)(2\pi - \arcsin 1)$

(9) $\sin(\arcsin \frac{1}{4})$
(10) $\cos(\arccos \frac{\sqrt{7}}{7})$
(11) $\arctan(\operatorname{tg} \sqrt{5})$
(12) $\arccos(\cos 12)$
(13) $\arcsin(\sin 100)$
(14) $\operatorname{tg}\left(2\arccos \frac{\sqrt{2}}{2} - \arcsin \frac{\sqrt{3}}{2}\right)$
(15) $\cos(3\arcsin \frac{\sqrt{3}}{2} + \arccos(-\frac{1}{2}))$
(16) $\sin(\arccos \frac{1}{3})$
(17) $\cos(\arcsin \frac{2}{3})$
(18) $\operatorname{tg}(\arcsin(-\frac{3}{4}))$
(19) $\cos(\arctan(-2))$
(20) $\sin(\arctan 3)$
(21) $\sin((\arcsin \frac{1}{5} + \arcsin \frac{3}{5}))$
(22) $\cos(\arcsin \frac{1}{\sqrt{3}} + \arccos(\frac{\sqrt{2}}{\sqrt{3}}))$
(23) $\sin(\arctan 2 - \operatorname{arc\,ctg} 3)$
(24) $\sin\left(\arccos(-\frac{1}{2}) - \arcsin \frac{1}{2}\right)$

Zad.4. Naszkicować wykresy podanych funkcji.

(1) $f(x) = |x^2 - 2x| + 2$
(2) $f(x) = ||x^2 - 6|x| + 8| - 4|$
(3) $f(x) = |2^{|x+1|} - 3|$
(4) $f(x) = ||\log_2(x-1)| - 1|$
(5) $f(x) = \frac{\sin \frac{x}{\pi}}{\pi}$
(6) $f(x) = |\operatorname{tg} |\frac{x}{2}||$
(7) $f(x) = |\sin |2x - \frac{\pi}{4}| + \frac{1}{2}|$
(8) $f(x) = |\cos \frac{2x-\pi}{4}| + 1$
(9) $f(x) = \arcsin \frac{|x|-2}{3}$
(10) $f(x) = \frac{\pi}{4} - \arccos \frac{1-x}{2}$

Zad.5. Zbadać parzystość podanych funkcji.

(1) $f(x) = \arcsin(12x) - 13$
(2) $f(x) = x \arcsin x$
(3) $f(x) = x \sin x + 2 \arcsin x$
(4) $f(x) = x \sin x + 2 \arccos x$
(5) $f(x) = \frac{\operatorname{tg} x}{x^3}$
(6) $f(x) = \frac{\arcsin x}{x^4}$
(7) $f(x) = \frac{\sin^3 x}{x}$
(8) $f(x) = \frac{\arcsin^5 x}{x^5}$
(9) $f(x) = \arcsin x \cdot \cos x$
(10) $f(x) = \arccos x \cdot \arcsin(x+1)$

Zad.6. Wykazać, że podane funkcje są różnowartościowe.

(1) $f(x) = \frac{2x+1}{x-3}$
(2) $f(x) = -x^5 - 16x - 1$
(3) $f(x) = 2^{x^3-1}$
(4) $f(x) = \sin \frac{1}{1+x^2}$ w $(0, \infty)$

Zad.7. Wykazać, że podane funkcje są rosnące.

(1) $f(x) = x^3 + 2x - 3$ (2) $f(x) = \ln(2^{3x} - 1)$ (3) $f(x) = e^{-\frac{1}{\log_5 x}}$

Zad.8. Wykazać, że podane funkcje są malejące.

(1) $f(x) = \ln \frac{1}{x^3+1}$ (2) $f(x) = -e^{2x^7+x^3}$ (3) $f(x) = -x^3 + 2x$ w $(1, \infty)$

Zad.9. Wyznaczyć funkcję odwrotną do danej.

(1) $f(x) = x^3 + 1$
(2) $f(x) = \frac{x-1}{x+2}$
(3) $f(x) = 2^{5x-3}$
(4) $f(x) = \log_4(x^5 + 2)$
(5) $f(x) = \sqrt[7]{x^4 + 1}$
(6) $f(x) = \sqrt[5]{2\ln(\operatorname{tg} x^3)}$, $D = \left(0, \sqrt[3]{\frac{\pi}{2}}\right)$
(7) $f(x) = \operatorname{arctan}(\log_5^3(e^x + 1))$
(8) $f(x) = \frac{\ln x}{\ln x+1}$
(9) $f(x) = \arcsin(\ln x)$
(10) $f(x) = \operatorname{arctan} \frac{1}{x}$
(11) $f(x) = \operatorname{ctg} \frac{\log_3 x+2}{\log_3 x-2}$
(12) $f(x) = \frac{\operatorname{tg} x+3}{\operatorname{tg} x-1}$, $D = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \setminus \left\{\frac{\pi}{4}\right\}$
(13) $f(x) = \frac{\operatorname{tg} x+3}{\operatorname{tg} x-1}$, $D = \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \setminus \left\{\frac{5\pi}{4}\right\}$
(14) $f(x) = \frac{\arcsin x-1}{\arcsin x+2}$
(15) $f(x) = \sqrt[3]{\frac{\sqrt[5]{\sin x}-2}{\sqrt[5]{\sin x}+2}}$, $D = \langle 0, \frac{\pi}{2} \rangle$