Granice ciągów

Zad.1. Zbadać monotoniczność podanych ciągów (a_n).

(1) $a_n = 2n^2 + 4n$
(2) $a_n = n^2 - 8n + 15$
(3) $a_n = \frac{n-1}{n+3}$
(4) $a_n = \frac{2+n}{2+n^2}$
(5) $a_n = \frac{n!-1}{n!+1}$
(6) $a_n = (-4)^{n+3}$
(7) $a_n = \frac{3^n}{n!}$

Zad.2. Zbadać, które z podanych ciągów są ograniczone z góry lub z dołu.

(1) $a_n = \frac{3n+1}{5n-2}$
(2) $a_n = \frac{2n-1}{n^2+5}$
(3) $a_n = \frac{\sqrt{\sqrt{n}+1}}{\sqrt[3]{\sqrt[3]{n}+2}}$
(4) $a_n = \operatorname{tg} \frac{1}{n}$
(5) $a_n = n \sin \frac{n\pi}{2}$
(6) $a_n = \left(1 - \frac{1}{n}\right)^n$
(7) $a_n = \sqrt[n]{2^n + n}$
(8) $a_n = \frac{(-1)^n n^2 + (-1)^{3n} n}{(-2)^n n^2 + (-2)^{3n} n}$
(9) $a_n = \frac{(-n)^3+1}{n+1}$
(10) $a_n = \frac{1}{\arcsin \frac{1}{n}}$

Zad.3. Obliczyć granice podanych ciągów (a_n).

(1) $a_n = \frac{7}{3n^2+1}$
(2) $a_n = \frac{5n^2-4n+3}{3n^2-5n+11}$
(3) $a_n = \frac{(n-2)(n-12)}{n^2-n+3}$
(4) $a_n = \frac{3n^3-n^2-2n-1}{4n^3+5n^2-1}$
(5) $a_n = \frac{2n^2(n^3-1)(n^3+1)}{5n^8+2n^2-1}$
(6) $a_n = \frac{n^5+3}{n^{10}+3}$
(7) $a_n = \frac{\frac{1}{2}n(3n+1)(4n+2)}{(2n-1)(n^2+1)}$
(8) $a_n = \frac{(2n+1)(n^2-7)}{(4n-1)(5n-1)}$
(9) $a_n = \frac{5n^3+n}{n^7-2}$
(10) $a_n = \frac{n^{12}+3n^2}{n^{10}+4}$
(11) $a_n = \frac{n^7-7n^6+1}{n^6+6n^5+7}$
(12) $a_n = \frac{4n^7-1}{7n^{14}+1}$
(13) $a_n = (\frac{6n-15}{3n+2})^4$
(14) $a_n = \left(\frac{3n^2+1}{3n^2-1}\right)^{12}$
(15) $a_n = (\frac{3n+7}{2n-3})^{35}$
(16) $a_n = \frac{\sqrt{n}}{3n-1}$
(17) $a_n = \frac{2\sqrt{n}+1}{2n^2-1}$
(18) $a_n = \frac{n}{\sqrt[5]{2n^5+3}}$

(19) $a_n = \frac{\sqrt[4]{n^4+n}}{3n+1}$

(20) $a_n = \frac{\sqrt[4]{n}-2}{\sqrt{n}+1}$

(21) $a_n = \frac{\frac{1}{2}n\sqrt{n}-n+1}{\frac{1}{3}n^2-1}$

(22) $a_n = \frac{-5\sqrt[4]{n}+3}{\sqrt{n}+\sqrt[4]{n}}$

(23) $a_n = n - \sqrt{n^2+1}$

(24) $a_n = n - \sqrt{n^2+2n}$

(25) $a_n = 5n - \sqrt{25n^2+1}$

(26) $a_n = \sqrt{n^2-n} - n$

(27) $a_n = \sqrt{n^2+n} - \sqrt{n^2-1}$

(28) $a_n = \frac{\sqrt{n+2}-\sqrt{n+1}}{\sqrt{n+1}-\sqrt{n}}$

(29) $a_n = \frac{2}{n-\sqrt{n^2+4n}}$

(30) $a_n = \frac{-1}{\sqrt{n^2+5n+5}-n}$

(31) $a_n = \frac{1}{3}n(\sqrt{n^2+n}-n)$

(32) $a_n = \sqrt{n^4+3n^2+1} - n^2$

(33) $a_n = \sqrt[3]{n^3+2} \cdot \sqrt[3]{n^3-2}$

(34) $a_n = \sqrt[3]{n^3+2} - \sqrt[3]{n^3-2}$

(35) $a_n = \sqrt{3 \cdot 4^{2n}-1} - \sqrt{5 \cdot 16^n+1}$

(36) $a_n = \frac{3^n+1}{2\cdot 3^n+1}$

(37) $a_n = \frac{2^n-3\cdot 5^n}{6\cdot 5^n-2}$

(38) $a_n = \frac{4^n+8\cdot 6^n}{2^n+7\cdot 6^n}$

(39) $a_n = \frac{5\cdot 2^n+2\cdot 5^n}{10^n}$

(40) $a_n = \frac{7^n-3\cdot 4^n}{2\cdot 3^n+4}$

(41) $a_n = \frac{n!-1}{n!+1}$

(42) $a_n = \frac{(n-1)!-(n+1)!}{(n-1)!+(n+1)!}$

(43) $a_n = \frac{2\cdot(n+1)!}{n!+2}$

(44) $a_n = \frac{\sqrt[n]{7}+\sqrt[n]{8}}{\sqrt[n]{n}+\sqrt[n]{\frac{1}{3}}}$

(45) $a_n = \frac{\sqrt[n]{2}-7}{\sqrt[n]{7}-2}$

(46) $a_n = \frac{\sqrt[n]{3}\cdot n}{n+11}$

(47) $a_n = \frac{3^{\sqrt{2n+1}}}{3^{\sqrt{2n}}}$

(48) $a_n = \frac{2^{\sqrt{n^2+n}}}{2^{\sqrt{n^2+3n}}}$

(49) $a_n = \log(n^2+7) - 2\log n$

(50) $a_n = \log(n^4+2n+1) - 2\log(n^2+1)$

(51) $a_n = 4\log(2n) - \log\frac{2n^2-1}{3} - \log(3n^2-1)$

(52) $a_n = \sqrt[n]{n^2+1}$

(53) $a_n = \sqrt[n]{\frac{1}{n}+\frac{\cos\pi n}{n^4}}$

(54) $a_n = \sqrt[n]{\left(\frac{2n^2-1}{n+2}\right)^3+31}$

(55) $a_n = \sqrt[n]{2^n+3^n}$

(56) $a_n = \frac{\sqrt[n]{4^n+5^n}}{\sqrt[n]{3^n+4^n}}$

(57) $a_n = \sqrt[n]{4^n+2^n+(\frac{1}{2})^n+(\frac{1}{4})^n}$

Zad.4. Obliczyć granice podanych ciągów (a_n), w których wzorach ogólnych pojawiają się sumy ciągu arytmetycznego lub geometrycznego.

(1) $a_n = \frac{4+7+10+...+(3n+1)}{2n^2-1}$

(2) $a_n = \frac{1+5+8+...+(4n-3)}{3n^2-1}$

(3) $a_n = \frac{-\frac{1}{3}n^2-2n}{3+6+...+3n}$

(4) $a_n = \frac{1+3+5+...+(2n-1)}{1+2+3+...+n}$

(5) $a_n = (\frac{1+2+...+n}{n+4})^2 - \frac{1}{4}n^2$

(6) $a_n = \frac{1+\frac{1}{3}+\frac{1}{3^2}+...+\frac{1}{3^n}}{1+\frac{1}{4}+\frac{1}{4^2}+...+\frac{1}{4^n}}$

(7) $a_n = \frac{\frac{1}{2}+\frac{1}{2^2}+\ldots+\frac{1}{2^n}}{-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\ldots+\frac{1}{3^n}}$

(8) $a_n = \frac{1-2+3-4+\ldots+(2n-1)-2n}{\sqrt{n^2+1}}$

Zad.5. Obliczyć granice ciągów o wyrazie ogólnym (a_n).

(1) $a_n = (1+\frac{2}{n})^{3n}$

(2) $a_n = (1-\frac{3}{4n})^{6n+1}$

(3) $a_n = (\frac{n+4}{n+2})^{n-1}$

(4) $a_n = \left(\frac{3n-1}{3n+4}\right)^{n+2}$

(5) $a_n = \frac{4(n+4)^n}{(n+3)^n}$

(6) $a_n = \frac{(n+1)^n}{(n+7)^n}$

(7) $a_n = (1-\frac{1}{n^2})^n$

(8) $a_n = (1+\frac{n+1}{n^2+2})^n$

(9) $a_n = \left(1-\frac{n+2}{n^2+1}\right)^{3n+1}$

(10) $a_n = (\frac{n^2-5n+4}{n^2+5n+4})^{2n-1}$

(11) $a_n = \left(\frac{2n^2+2n+1}{2n^2-3n+2}\right)^{3n+1}$

(12) $a_n = (\frac{n^3+n-2}{n^3+n+2})^{n^2-1}$

(13) $a_n = \left(\frac{n^3+n^2-2}{n^3-n^2-2}\right)^{n+1}$

(14) $a_n = \left(\frac{n^3-n^2-2}{n^3+n^2+2}\right)^{n^2+1}$

(15) $a_n = (\frac{n}{5n-1})^n$

(16) $a_n = (\frac{6n+1}{n})^n$

(17) $a_n = n(\ln(n+7)-\ln(n+2))$

(18) $a_n = 2n\,(\ln(3n+1)-\ln(3n))$

(19) $a_n = \sqrt{n}(\ln(\sqrt{n}+3)-\ln(\sqrt{n}))$

(20) $a_n = 3n(\ln(2n+5)-\ln(2n+1))$

(21) $a_n = 2n^3(\ln(n^3+4)-\ln(n^3+1))$

Zad.6. Obliczyć granice ciągów o podanych wyrazach ogólnych a_n.

(1) $a_n = \frac{i-1}{n^2}$

(2) $a_n = \frac{in^2-3}{n^4+n-1}$

(3) $a_n = \frac{(in-2)(in+2)}{(n-2i)(n+2i)}$

(4) $a_n = \frac{(3n^2+i-1)^2}{(in+2)^2}$

(5) $a_n = \frac{5^n-i^n}{6^n+i^n}$

(6) $a_n = \left(\frac{2i}{3+i}\right)^n$

(7) $a_n = |n+2i|-|n-i|$

(8) $a_n = |n+i|-|n-i|$

(9) $a_n = |i+\frac{i^n}{n}|-|i-\frac{i^{n^2}}{n^2}|$

(10) $a_n = \frac{n+i}{|n-i|}$

Zad.7. Dla jakich wartości parametru p ciąg o wyrazie ogólnym $a_n = \frac{pn+1}{(p-1)n-2}$ jest:

(1) zbieżny do granicy 2,

(2) zbieżny do granicy $\frac{1}{2}$,

(3) rozbieżny do $+\infty$,

(4) zbieżny do granicy 0?

Zad.8. Dany jest ciąg zadany wzorem $a_n = \frac{(p^2-p)n^2+p\cdot n-2}{(p-1)n^2+2p\cdot n+1}$, gdzie p jest parametrem. Jakie wartości może przyjmować granica tego ciągu?

Zad.9. Obliczyć granice ciągów o podanych wzorach ogólnych.

(1) $a_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{(n-1)n}$ (2) $a_n = (1 - \frac{1}{2^2})(1 - \frac{1}{3^2}) \ldots (1 - \frac{1}{n^2})$